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A Regime Switching for dynamic conditional correlation and GARCH: Application to agricultural commodity prices and market risks

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Abstract. Time varying correlations are often estimated with dynamic conditional correlation, generalized autoregressive conditional heteroskedasticity (DCC-GARCH) models which are based on a linear structure in both GARCH and DCC parts. In this paper, a Markov regime-switching dynamic conditional correlation, generalized autoregressive conditional heteroscedasticity (MS-DCC-GARCH) model is proposed in order to capture the time variations and structural breaks in both GARCH and DCC processes. The parameter estimates are driven by first order Markov chain. We provide simulation study to examine the accuracy of the model and apply it for empirical analysis of the dynamic volatility correlations between commodity prices and market risks. The proposed model is clearly preferred in terms of likelihood, Akaike information criterion (AIC), and likelihood ratio test.

Keywords:

1 Introduction

It is well known that there exists co-movement of financial volatilities more or less closely over time across assets and markets. Hence it is important to take into account this time varying co-movement. To date, there are various models capable of measuring the dynamic volatility and correlations between market risk and other assets; for instance, the multivariate GARCH (generalized autoregressive conditional heteroskedasticity) model. This model estimates the covariance matrix between the assets by extending a univariate GARCH into a multivariate GARCH model. To extend the univariate to multivariate GARCH under the dynamic context, the dynamic copula and dynamic conditional correlation (DCC) is proposed to decompose the conditional covariance matrix into a conditional standard deviation matrix and a conditional correlation matrix, for example dynamic conditional correlation DCC-GARCH (Engle, 2002) and dynamic copula-GARCH (Patton, 2006). Although these dynamic linear approaches have been undertaken for time varying variance and correlations, but with limitations; for example, there are not take into account the nonlinear structure in dynamic correlations. Empirical evidences suggested that the behavior of economic and financial time series may exhibit different patterns over time (see, Billio and Caporin, (2005), Pastpipatkul, et al. 2016)). Hence, instead of using the linear conditional variance, it has become more natural to extend the linear model into the model that properly reflects those structural change patterns. As a consequence, the Markov Switching model was extended to those linear models (see, Billio and Caporin, (2005),Da Silva Filho, Ziegelmann, and Dueker, 2012). However, many practitioners; in fact, have recently paid more attention to considering more than two assets. Therefore, the Markov Switching dynamic bivariate copula of Da Silva Filho, Ziegelmann, and Dueker (2012) has dissatisfied many practitioners who worked on multivariate volatilities and correlations.

In this work, we focus on the Markov Switching dynamic correlation GARCH (MS-DCC-GARCH) model of Billio and Caporin, (2005). The model has the flexibility of univariate GARCH but not the complexity of conventional multivariate GARCH. The model is estimated in two steps: firstly, a series of univariate GARCH parameters are estimated and then dynamic conditional correlation parameters are estimated in the second step. In other words, the parameters to be estimated in the correlation and GARCH processes are independent (Engle, 2002). In the extension of Billio and Caporin (2005), the model was modified to allow only a dynamic correlation to switch between two or more regimes. Therefore, they restricted the regime dependent structure only to the DCC parameters excluding GARCH parameters. They mentioned that the model in which all parameters are allowed to switch might lead to unstable results and difficult to reach the global maximum likelihood due to the large number of switching parameters. However, we are still concerned about the consistency and asymptotic normality of this two-step estimation. We need to have a reasonable regularity conditions to ensure that the first step will ensure consistency of the second step. The maximum of the second step will be a function of the first step parameter estimates, so these two steps need to be consistency. To overcome this problem, our study modifies the likelihood function of MS-DCC-GARCH of Billio and Caporin (2005) by incorporating a likelihood of DCC to likelihood of Multivariate GARCH in order to do a one-step maximum likelihood and also modifies possible change in all parameters in both GARCH and DCC parts.

The main objective of this research is to extend MS-DCC GARCH of Billio and Caporin (2005) by allowing the parameters in mean, variance and correlation equations to switch across different regimes or to be state dependent according to the first order Markov process. This means that all parameters are governed by a state variable S_t which is assumed to evolve according to S_{t-1} with transition probability. Then the one step maximum likelihood is constructed as an estimator of the model. Subsequently, we apply our model to study the dynamic volatility correlations between commodity prices and market risks.

Measuring the dynamic volatility correlations between commodity prices and market risks has an important implication for economic growth and investment. The increased integration between financial markets and the commodity markets provides an alternative way for investors to invest, diversify and hedge their portfolio's risks. To manage the portfolio's risks, investors and regulators need to take into account the corre-

lation between assets across international financial markets. This study therefore aims to investigate the relationship between the agricultural commodity prices in the light of the risk perceptions and uncertainty that led to the global financial crisis. We consider the indicators of the perceived global risk and global market conditions, namely, the Volatility (VIX) index and Credit Default Swaps (CDS). These two indicators are the benchmark proxy for the risk perception of investors (Gğzgör, Kablamac, 2014). As a consequence, in this study, we will show the performance of our model to measure the dynamic correlation and volatility among agricultural commodities including corn, wheat, and soybean; and global risk including VIX index and CDS.
The organization of this paper is as follows. Section 2 describes the methodologies used in the model. Section 3 conducts a simulation study to evaluate and illustrate our model. Section 4 employs the proposed model to investigate the dynamics between three agricultural commodities and market risks. Finally, we provide concluding remarks in Section 5.

2 Methodology

In this study, our concern is on the non-linear behavior of financial data which entitles the conventional DCC-GARCH model not appropriate for describing the correlation and volatility of the returns of the assets. To deal with this problem, a Markov Switching dynamic conditional correlation GARCH (MS-DCC-GARCH) is considered in this study. We generalizes the MS-DCC-GARCH model of Billio and Caporin (2005), Pelletier (2006) and Chen (2009) in that the parameters to be estimated in the GARCH and DCC processes are dependent and allowed to vary across regimes.

2.1 Univariate GARCH(1,1) Model

In the application study, the GARCH (1,1) model is sufficient to capture the volatility clustering in the data (Bollerslev, Chou and Kroner, 1992).Thus, the GARCH(1,1) specification for the volatility spillover model follows Bollerslev (1986) and is specified as

$$r_{i,t} = u_i + \varepsilon_{i,t} = u_i + \sqrt{\sigma^2_{i,t} a_{i,t}}, \tag{1}$$

$$\sigma^2_{i,t} = \alpha_{i,0} + \alpha_{i,1}\varepsilon^2_{i,t-1} + \beta_{i,1}\sigma^2_{i,t-1}, \tag{2}$$

where $r_{i,t}$ is return of asset i at time t, u_i is constant term, $\varepsilon_{i,t}$ is the error term composed of a sequence of iid standardized residual with normal distribution ,$a_{i,t}$, and conditional variance, $\sigma^2_{i,t}$. Bollerslev (1986) provides a systematic framework for asset volatility modeling which has proved particularly valuable in time series with time-varying variance, $\sigma^2_{i,t}$ as presented in Eq.2. Some restrictions are set in this model as follows: $\alpha_0 > 0, \alpha_1 \geq 0, \beta_1 \geq 0$, and $(\alpha_1 + \beta_1) < 1$. The latter constraint on $\alpha_1 + \beta_1$ implies that the unconditional variance of $r_{i,t}$ is finite.

2.2 DCC-GARCH(1,1) Model

The DCCGARCH model is the extension of GARCH model with the purpose to capture multivariate volatility (see, Eagle, 2002). The advantage of the DCC model is that we can estimate the time-varying correlations between multi dimensional returns instead of a constant correlation (CCC model of Bollerslev (1990)). Considering n-dimensional return series R_t we can write multivariate DCC-GARCH as:

$$R_t = U + \sqrt{H_t^2 A_t}, \tag{3}$$

$$H_t = D_t Z_t D_t, \tag{4}$$

where U is $n \times 1$ vector of constants, A_t is $n \times T$ matrix of standardized residuals. H_t is $n \times T$ matrix of conditional variances from univariate GARCH model in section 2.1. Z_t is the conditional correlation matrix of the standardized residuals $\varepsilon_{i,t}$. $D_t = \text{diag}(\sigma_{1,t}^{1/2}, \cdots, \sigma_{n,t}^{1/2})$ is the diagonal matrix composed of the conditional variance of n returns. Z_t is given by:

$$Z_t = Q_t^{*-1/2} Q_t Q_t^{*-1/2}, \tag{5}$$

then, the DCC equation can be specified by

$$Q_t = (1 - \theta_1 - \theta_2)\overline{Q} + \theta_2 Q_{t-1} + \theta_2 \varepsilon_{t-1} \varepsilon_{t-1}', \tag{6}$$

where, $\overline{Q} = \frac{1}{T} \sum_{t=1}^{T} \varepsilon_{t-1} \varepsilon_{t}'{}_{1}$ is the unconditional covariance matrix of the standardized residuals , $\varepsilon_{t-1} = \{\varepsilon_{1,t-1}, \cdots, \varepsilon_{n,t-1}\}$ and Q_t^* is a diagonal matrix with the square root of the diagonal elements of Q_t. The coefficients θ_1 and θ_2 must satisfy: $0 \le \theta_1 + \theta_2 < 1$.

2.3 Markov switching DCC-GARCH(1,1)

As we mentioned before, the study aims to allow all parameters to switch across states or regimes. Thus, the general form of the Markov switching DCC- GARCH(1,1) model can be written as

$$R_t = U + \sqrt{H_{S_t}} A_{S_t}, \tag{7}$$

$$H_{S_t} = D_{S_t} Z_{S_t} D_{S_t}, \tag{8}$$

$$Z_{S_t} = Q_{S_t}^{*-1} Q_{S_t} Q_{S_t}^{*-1}, \tag{9}$$

$$Q_{S_t,t} = (1 - \theta_{1,S_t} - \theta_{2,S_t})\overline{Q}_{S_t,t} + \theta_{2,S_t} Q_{S_t,t-1} + \theta_{2,S_t} \varepsilon_{S_t,t-1} \varepsilon_{S_t,t-1}', \tag{10}$$

where Eq.(7) and Eq.(8) are the mean and variance equations, respectively, and they are allowed to switch across regime. The feature of the Markov switching model is the estimated parameters in mean, variance and correlation equations, Eq.(7), Eq.(8) and Eq.(9), can switch across different regimes or are state dependent according to the first order Markov process. This means that all parameters are governed by a state variable S_t which is assumed to evolve according to S_{t-1} with transition probability, p_{ij}, thus

$$P(S_j = 1 | S_{t-1} = i) = p_{ij}, \sum_{j=1}^{J} p_{ij} = 1, \text{ for } i = 1, \cdots, J. \tag{11}$$

In this study, two regimes are considered thus, the first order Markov process could be written as:

$$\begin{array}{l} P(S_j = 1 | S_{t-1} = 1) = p_{11} \\ P(S_j = 1 | S_{t-1} = 2) = p_{12} \\ P(S_j = 2 | S_{t-1} = 1) = p_{21} \\ P(S_j = 2 | S_{t-1} = 2) = p_{22} \end{array} \quad (12)$$

Let $\Theta = (U, \alpha_{S_t}, \beta_{S_t}, \theta_{1,S_t}, \theta_{2,S_t}, p_{11}, p_{22})$ be the vector of model parameters. From Eqs. (7-10), according to Eagle (2002), we can write the likelihood function of DCC-GARCH(1,1) as

$$L(\Theta) = L_v(\theta) \cdots L_Z(\alpha, \beta), \quad (13)$$

where $L_v(\theta)$ and $L_Z(\alpha, \beta)$ are volatility part and a correlation part, respectively. Thus, we can rewrite our likelihood of Markov switching DCC-GARCH(1,1) as

$$L(\Theta_{S_t}) = L_v(\theta_{S_t}) \cdots L_Z(\alpha_{S_t}, \beta_{S_t}), \quad (14)$$

where

$$L_v(\theta_{S_t}) = \prod_{t=1}^{T} \frac{1}{(2\pi)^{k/2}} \exp\{-\frac{1}{2} \varepsilon_{S_t}^T \varepsilon_{S_t}\}, \quad (15)$$

and

$$L_R(U_{S_t}, \alpha_{S_t}, \beta_{S_t}) = \prod_{t=1}^{T} \frac{1}{(2\pi)^{k/2} |Z_{S_t}|} \exp\{-\frac{1}{2} e_{S_t}^T Z_{S_t}^{-1} e_{S_t}\}. \quad (16)$$

Therefore, we will denote the full likelihood of Markov switching DCC-GARCH(1,1) with J regimes for k assets by

$$L(\Theta_{S_t} | r_{1,t}, \cdots, r_{kt}) = \sum_{j=1}^{J} \left(\prod_{t=1}^{T} \prod_{k=1}^{K} L(\Theta_{S_t} | r_{kt}) (Pr(S_t = j | \Theta_{S_t})) \right) \quad (17)$$

2.4 Hamilton Filter

According to Eq.(17), the filtered probability $Pr(S_t = j | \Theta_{t,S_t})$ is an important process to assign estimated coefficient and variance parameters into two different regimes. The most famous filtering approach is introduced in Hamilton (1989) called the Hamilton Filter. Suppose, we assume two- regime Markov switching DCC-GARCH, then the Hamilton filter is executed according to the following algorithm.

1. Given an initial guess of transition probabilities which are the probabilities P of switching between regimes, the transition probabilities of two regimes are

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \quad (18)$$

2. Update the transition probabilities of each state with the past information including the parameters in the system equation, $\Theta_{S_{t-1}}$ and P, for calculating the likelihood function in each state at time t. After that, the probabilities of being in each state are to be updated by the following formula

$$Pr(S_t = j)|\theta_{S_t}) = \frac{f(r_{1,t},\cdots,r_{kt}|S_t = j\Theta_{S_{t-1}})Pr(S_t = j|\Theta_{S_{t-1}})}{\sum_{j=1}^{J} f(r_{1,t},\cdots,r_{kt}|S_t = j\Theta_{S_{t-1}})Pr(S_t = j|\Theta_{S_{t-1}})} p_{jj} \tag{19}$$

where $f(r_{1,t},\cdots,r_{kt}|S_t = j\Theta_{S_{t-1}})$ is the full likelihood function in Eq.(10) and $Pr(S_t = j|\Theta_{S_{t-1}})$ is a filtered probability.

3. Iterate steps 1 and 2 for $t = 1,\cdots,T$.

3 . Simulation study.

In this section, we examine the accuracy of the proposed model by conducting a simulation study. To this end, we consider a two-regime MS-DCC-GARCH with 3 variables as follows:

$$\left.\begin{aligned} \sigma^2_{1,t} &= 0.001 + 0.02\varepsilon^2_{1,t-1} + 0.70\sigma^2_{1,t-1}, \\ \sigma^2_{2,t} &= 0.003 + 0.05\varepsilon^2_{2,t-1} + 0.70\sigma^2_{2,t-1}, \\ \sigma^2_{3,t} &= 0.004 + 0.04\varepsilon^2_{3,t-1} + 0.80\sigma^2_{3,t-1}, \\ Q_{S_t,t} &= (1 - 0.01 - 0.95)\overline{Q}_{S_t,t} + 0.95Q_{S_r,t-1} + 0.95\varepsilon_{S_{t,t-1}}\varepsilon'_{S_{t,t-1}}, \end{aligned}\right\} S_t = 1$$

$$\left.\begin{aligned} \sigma^2_{1,t} &= 0.009 + 0.07\varepsilon^2_{1,t-1} + 0.90\sigma^2_{1,t-1}, \\ \sigma^2_{2,t} &= 0.002 + 0.02\varepsilon^2_{2,t-1} + 0.95\sigma^2_{2,t-1}, \\ \sigma^2_{3,t} &= 0.0006 + 0.15\varepsilon^2_{3,t-1} + 0.80\sigma^2_{3,t-1}, \\ Q_{S_t,t} &= (1 - 0.02 - 0.60)\overline{Q}_{S_t,t} + 0.60Q_{S_r,t-1} + 0.60\varepsilon_{S_{t,t-1}}\varepsilon'_{S_{t,t-1}}, \end{aligned}\right\} S_t = 2 \tag{20}$$

In this simulation, the model is simulated based on a normal distribution. Moreover, we consider three sample sizes :$T = 1000$, $T = 2000$, and $T = 3000$. We randomly the state variable S_1 from a first-order Markov process taking values $\{1,2\}$ and set $p_{11} = 0.90$ and $p_{22} = 0.85$.

From the simulation results in Table 1, we can see that the estimated parameters are close to their true values in the first column. In addition, when the number of sample is larger, the model performs more accurate. Furthermore, the result of filtered probabilities is also plotted in Figure 1 and we can see that the Hamilton Filter algorithm works well in capturing the hidden state in the data since the estimated smoothed probabilities (red line) are close to true probabilities (blue line). Thus, the simulation results show that the proposed model is accurate and efficient.

Table 1. Simulation results

Parameter	TRUE	Estimated		
		T=1000	T=2000	T=3000
$\alpha_{01,S_t=1}$	0.001	0.0006	0.0005	0.0006
$\alpha_{11,S_t=1}$	0.02	0.001	0.0015	0.0021
$\beta_{11,S_t=1}$	0.7	0.8008	0.7998	0.8098
$\alpha_{02,S_t=1}$	0.003	0.002	0.0017	0.002
$\alpha_{12,S_t=1}$	0.005	0.0003	0.0027	0.005
$\beta_{12,S_t=1}$	0.7	0.7998	0.7994	0.8023
$\alpha_{03,S_t=1}$	0.004	0.0047	0.004	0.0042
$\alpha_{13,S_t=1}$	0.04	0.0001	0.0047	0.004
$\beta_{13,S_t=1}$	0.8	0.7997	0.7996	0.8086
$\alpha_{01,S_t=2}$	0.009	0.0257	0.0241	0.0275
$\alpha_{11,S_t=2}$	0.07	0.0538	0.0548	0.0614
$\beta_{11,S_t=2}$	0.9	0.8896	0.8878	0.8882
$\alpha_{02,S_t=2}$	0.005	0.0467	0.0481	0.0477
$\alpha_{12,S_t=2}$	0.02	0.016	0.0161	0.0155
$\beta_{12,S_t=2}$	0.95	0.8984	0.8873	0.8865
$\alpha_{03,S_t=2}$	0.006	0.0084	0.0068	0.0078
$\alpha_{13,S_t=2}$	0.15	0.043	0.206	0.1445
$\beta_{13,S_t=2}$	0.8	0.8936	0.8972	0.904
$\theta_{1,S_t=1}$	0.01	0.0078	0.0079	0.0079
$\theta_{2,S_t=1}$	0.95	0.9213	0.9203	0.9205
$\theta_{1,S_t=2}$	0.02	0.0149	0.013	0.0146
$\theta_{2,S_t=2}$	0.6	0.5005	0.5	0.5002
P_{11}	0.9	0.8943	0.8953	0.8944
P_{22}	0.85	0.8794	0.8751	0.8682

Source : calculation

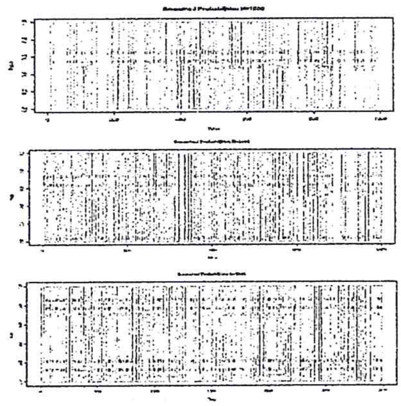

Fig. 1. Simulated smoothed probabilities and estimates probabilities (blue =true line, red =estimates)

4 Real data estimation

4.1 Data Description

Table 2. Descriptive statistics

	Corn	Wheat	Soybean	VIX	CDS
Mean	0.0004	0.0002	0.0003	−0.0001	0.0002
Median	0.001	−0.0002	0.0018	−0.0041	−0.0025
Maximum	0.0881	0.0693	0.1013	0.3394	0.1661
Minimum	−0.1104	−0.0738	−0.115	−0.2416	−0.1284
Std. Dev.	0.0195	0.02	0.0173	0.063	0.0307
Skewness	−0.4234	0.1337	−0.8318	0.6309	0.4223
Kurtosis	6.1143	3.8794	8.9886	5.7896	5.9205
Jarque-Bera	271.6729	22.0375	1007.616	244.514	241.0787
Probability	0	0	0	0	0
ADF-Prob.	0	0	0	0	0

Source : calculation

We examine a systematic relationship between agricultural commodity prices consisting of Corn, Wheat, and Soybean futures. We also consider the role of the global market risks over the period from 1/4/2005 through 31/3/2017, covering 625 observations. The frequency used in this study is the weekly closing prices. The data set is obtained from Thomson Data stream database. To measure the effect of global market risks, we use the VIX index of the Chicago Board Options Exchange (CBOE) and the Credit Default Swaps (CDS) in the US market. Then, all weekly observations are converted into returns in a standard method as log differences.

The descriptive statistics of return series are shown in Table 2. We can see that the returns show similar characteristics. All returns exhibit high kurtosis but small skewness. This indicates that these returns have a small tail. Furthermore, the normality of returns is strongly rejected by the normality Jarque-Bera test, with probability=0.000. In addition, the result of the Augmented Dickey-Fuller (ADF) test suggests that the null hypothesis of the unit root test can be rejected and that all the variables are stationary at level under the 1% confidence level.

5 Empirical findings

5.1 MS-DCC-GARCH(1,1) results

Table 3 reports two-regime MS-DCC-GARCH (1, 1) estimation results for all pairs to examine the regime dependent dynamic conditional correlation and variance. Note that, our model allows the GARCH and DCC parameters to vary across the regimes. It is clear from the table that almost all parameters are highly significant at 1%. The degree

Table 3. MS-DCC-GARCH parameters estimated

			Regime 1		
GARCH					
	Corn	**Wheat**	**Soybean**	**VIX**	**CDS**
$\alpha_{0,S_t=1}$	0.0001***	0.0002	0.0001***	0.0015***	0.0003***
	(0.00001)	(0.431)	(0.00001)	(0.00001)	(0.00001)
$\alpha_{1,S_t=1}$	0.0554	0.0903	0.1901***	0.0658	0.1545***
	(0.1450)	(0.184)	(0.00001)	(0.4953)	(0.00001)
$\alpha_{1,S_t=1}$	0.8818***	0.8156***	0.8098***	0.8561***	0.8367***
	(0.00001)	(0.0001)	(0.00001)	(0.00001)	(0.00001)
Unconditional variance	0.9372	0.9059	0.9999	0.9219	0.9912
			Regime 2		
GARCH					
	Corn	**Wheat**	**Soybean**	**VIX**	**CDS**
$\alpha_{0,S_t=2}$	0.0001***	0.0002	0.0001***	0.0015***	0.0003***
	(0.00001)	(0.431)	(0.00001)	(0.00001)	(0.00001)
$\alpha_{0,S_t=2}$	0.0554	0.0903	0.1901***	0.0658	0.1545***
	(0.1450)	(0.184)	(0.00001)	(0.4953)	(0.00001)
$\alpha_{0,S_t=2}$	0.8818***	0.8156***	0.8098***	0.8561***	0.8367***
	(0.00001)	(0.0001)	(0.00001)	(0.00001)	(0.00001)
Unconditional variance	0.9372	0.9059	0.9999	0.9219	0.9912
DCC Regime 1		0.1405***		0.8197***	
		(0.00001)		(0.0001)	
DCC Regime 2		0.0296***		0.9184***	
		(0.0048)		(0.00001)	
		Transition parameter			**Duration**
p_{11}		0.8513***			6.7276
		(0.0001)			
p_{22}		0.8172***			5.4689
		(0.0001)			
Criterion			**DCC-GARCH**	**MS-DCC-GARCH**	**MS-DCC-fixed GARCH**
AIC			−12016.42	−12998.54	−12132.35
log-likelihood			6025.221	6535.2703	6087.174
	1) LR-test (H0:1 Vs Ha: 2)				P-value = 0.0000
	2) LR-test (H0:1 Vs Ha: 3)				P-value = 0.0000
	3) LR-test (H0:2 Vs Ha: 3)				P-value = 1.0000

Notes: P values are in parentheses. Parameter estimates are based on the MS-DCC (1,1)-GARCH(1,1) model.

of volatility persistence for the model can be obtained by summing ARCH, α_{1,S_t}, and GARCH , β_{1,S_t}, parameters. Different from the previous MS-DCC-GARCH models, our model distinguishes the volatility persistence into two regimes. Different results have been obtained from these parameters. We find that the sum of α_{1,S_t} and β_{1,S_t} in regime 1 is not close to 1 for all returns and also lower than in regime 2, indicating that volatility is likely to be high in regime 2.The volatility persistence coefficients measured by $\alpha_{1,S_t} + \beta_{1,S_t}$ in the GARCH specification are respectively 0.8064, 0.6433, 0.7479, 0.8350, and 0.8914 for the the returns of corn futures, wheat futures, soybean futures, VIX, and CDS variables in regime 1 and respectively 0.9372, 0.9059, 0.9999, 0.9219, and 0.9912 in regime 2. These findings lead us to interpret regime 1 as low volatility regime while regime 2 as high volatility regime. Then, considering the regime dependent conditional correlations, the parameters θ_{1,S_t} and θ_{2,S_t} are highly significant in both regimes. Therefore, there are significant correlations among the returns in both regimes. The estimates $\theta_{1,S_t} + \theta_{2,S_t}$ for across the regimes are not quite different, the low and high volatility regimes are characterized by different dynamic correlation structures. Indeed, the sums $\theta_{1,S_t} + \theta_{2,S_t}$ are 0.9602 (0.9480) for the low (high) volatility regime. The findings suggest that the low correlation is associated with the high volatility regime and vice versa regarding the relationship between agricultural commodities and market risks. Da Silva Filho (2013) and Pathairat.et. al. (2016), have found that the conditional correlation during market upturns is less than that during market downturns. Thus, this confirms that the high correlation mostly exists in the market downturn or low volatility regime and the low correlation mostly exists in the market upturn or high volatility regime.

The persistence and regime properties of the these returns as captured by the estimates of the MS-DCC-GARCH parameters show similar analogous features in terms of transition probabilities, ergodic (regime) probabilities, and duration in each regime. In the Table 3. We denote the probabilities $Pr(S_t = 1|S_{t-1} = 1)$ by p_{11} and $Pr(S_t = 2|S_{t-1} = 2)$ by p_{22}. We can notice that both of the regimes are persistent because of the high values obtained for the probabilities $p_{11} = 0.8513$ and $p_{22} = 0.8172$. We typically observe that the agricultural commodities, VIX and CDS have a high probability to spend much of the time in the low volatility regime, resulting in higher duration estimates for the low volatility regime compared to the high volatility regime. The duration of the low volatility regime is 6.7276 weeks while the that of the high volatility regime is 5.4689 weeks.

The estimated MS-DCC-GARCH (1,1) model also produces the probabilities of two regimes for the period from 2006 to 2016. In this section, we plot only the low volatility regime probabilities for all the returns presented in Figure 2. Based on the results, it is evident that the volatility of all returns has mostly taken place in the low volatility regime, except for the period from 2007 to 2010 (the blue-dashed line). This period corresponds to the Global financial crisis in 2007-2008. Our findings confirm the high volatility of all returns during the financial crisis. This period also coincided with the expansion of the quantitative easing of the US and the announcement of the Federal Reserve (FED) that they had decided to launch a new $40 billion a month bond purchase program of the agency Mortgage Back Securities (MBS). Therefore, we expect that these policies would probably put the high pressure directly to VIX and CDS and

thereby pushing the high volatility in this regime.
Furthermore, what is interesting is to compare the log-likelihood and AIC of the different models. In this study, we examine the performance of our model MS-DCC-GARCH(1,1) with regime switching in both GARCH and DCC parts, by comparing with two other conventional models namely MS-DCC-GARCH with regime switching only DCC part of Billio and Caporin (2005) and DCC-GARCH with no regime switching in both GARCH and DCC parts of Eagle (2002). According to the results in Table 3, we observe that our model performs slightly better than the two conventional models since we obtain the lowest AIC and the highest log- likelihood. However, Billio and Caporin (2005) mentioned that it is not easy to compare the best fit model through log-likelihood value, but they proposed to compare the models using the Likelihood ratio test statistics. The null hypothesis of restricted model (H0) is tested against unrestricted model (Ha). The results of LR-test statistics reject the first two tests and accept the third one. This means that our model cannot be rejected by LR-test.

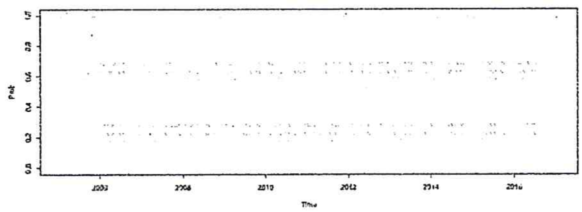

Fig. 2. Smoothed probabilities of low volatility regime

5.2 Volatility of Returns

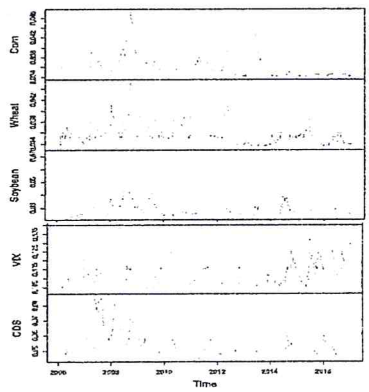

Fig. 3. Smoothed Volatilities

In this section, the volatilities of these returns are plotted in Figure 2. As our model is two-regime MS-DCC-GARCH (1,1), two-regime conditional volatilities are obtained from the GARCH process for each regime. Following Gray (1996), the expected conditional volatility was proposed to present the expected conditional volatility of returns. In this case, we can compute the expected conditional volatility as

$$E\left(\sigma_{i,S_t}^2\right) = \sum_{S_t=j}^{2}\left[\left(\alpha_{i,0,S_t} + \alpha_{i,1,S_t}\varepsilon_{i,t-1,S_t}^2 + \beta_{i,1,S_t}\right)\cdot\left(Pr(S_t = j|\Theta_{S_t})\right)\right] \tag{21}$$

Figure 2 illustrates the expected conditional volatility of all returns. In general, we can observe that the expected conditional volatilities of corn, wheat, soybean, and CDS are relatively high during 2007-2008, corresponding to the period of global financial crisis as we mentioned before. When we compare the value of volatility, we find that CDS returns present the highest volatility while VIX present the lowest volatility along our sample period.

5.3 Smoothed correlation of returns

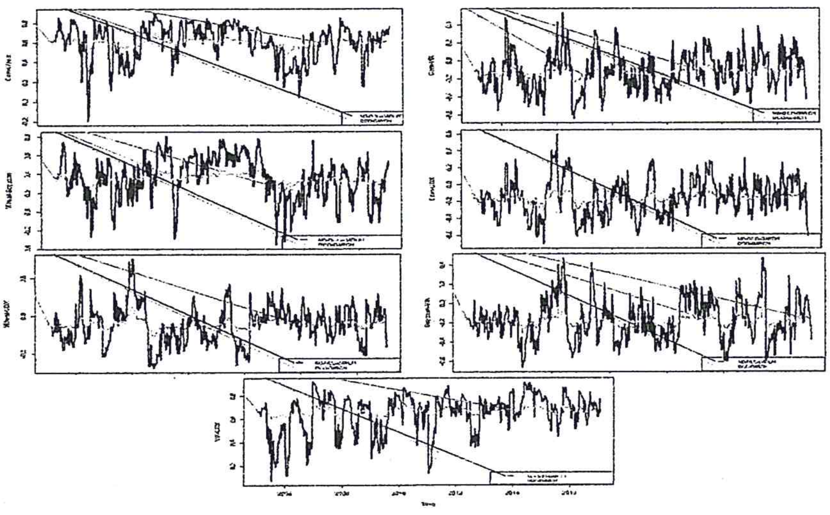

Fig. 4. Time-varying smoothed correlation between variables

Similar to the expected conditional volatility; in this section, we plot the expected dynamic conditional correlations from our model as well as the dynamic conditional correlations obtained from one-regime DCC-GARCH model. Figure 4 provides an evidence on the conditional correlation obtained from the DCC-GARCH(1,1) model (dash lines) versus our conditional expected correlation obtained from the MS-DCC-GARCH(1,1) model (solid lines). By visually evaluating Figure 4, it is apparent that forecasting from the MS-DCC-GARCH(1,1) is too noisy to represent the correlation when compared to DCC-GARCH(1,1). The differences between one-regime and two-regime models allow us to recognize the advantages of using the structural change detecting method. To be clearer, Figure 4 shows a high volatile correlation during the period 2008-2010, but a less volatile after 2012. This indicates that the Markov Switching approach yields higher volatilities and may better describe the characteristics in real empirical study.

6 Conclusions and Future research

In this study, we introduce a generalization of Markov Switching dynamic conditional correlation GARCH (MS-DCC-GARCH) of Billio and Caporin (2005) by allowing for Markov switches in both GARCH and DCC equations. The transitions between regimes are governed by a first order Markov chain. We also present a restricted version of our model where the changes across volatilities and correlations in a given regime are proportional. According to Hamilton (1989), we employ Hamilton filter algorithm for model estimation in order to filter the GARCH and DCC equations into two regimes. In order to break the curse of the consistency and asymptotic normality of this estimator. We construct the full maximum likelihood of MS-DCC-GARCH and employ a one-step estimation procedure to estimate all unknown parameters of our model. We then propose a simulation study to examine the accuracy of our model and find that our model and estimation are accurate and efficient.
An application of this model ia on investigating the dynamic correlations among three major agricultural commodity prices and two market risks. The comparison of our regime switching model with the DCC model of Engle (2002) and with the MS-DCC-GARCH of Billio and Caporin (2005) shows that our model has a better performance. An interesting aspect of our regime switching model is that we obtain a weak and strong persistence in the Markov chain, which produces both high and less volatility of dynamic correlations along the sample period.

References

1. Billio, M., & Caporin, M. (2005). Multivariate Markov switching dynamic conditional correlation GARCH representations for contagion analysis. Statistical Methods & Applications, 14(2), 145-161.
2. Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. Journal of econometrics, 31(3), 307-327.
3. Bollerslev, T. (1990). Modelling the coherence in short-run nominal exchange rates: a multivariate generalized ARCH model. The review of economics and statistics, 498-505.
4. Bollerslev, T., Chou, R. Y., & Kroner, K. F. (1992). ARCH modeling in finance: A review of the theory and empirical evidence. Journal of econometrics, 52(1-2), 5-59.
5. Chen, C. W., Lee, S., & Chen, S. Y. (2016). Local non-stationarity test in mean for Markov switching GARCH models: an approximate Bayesian approach. Computational Statistics, 31(1), 1-24.
6. Chollete, L., Heinen, A., & Valdesogo, A. (2009). Modeling international financial returns with a multivariate regime-switching copula. Journal of financial econometrics, 7(4), 437-480.

7. Gzgr, G., Kablamac, B. (2014). The linkage between oil and agricultural commodity prices in the light of the perceived global risk.
8. Gray, S. F. (1996). Modeling the conditional distribution of interest rates as a regime-switching process. Journal of Financial Economics, 42(1), 27-62.
9. Pelletier, D. (2006). Regime switching for dynamic correlations. Journal of econometrics, 131(1), 445-473.
10. Chen, R. (2009). Regime switching in volatilities and correlation between stock and bond markets.
11. Engle, R. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. Journal of Business & Economic Statistics, 20(3), 339-350.
12. Hamilton, J. D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. Econometrica: Journal of the Econometric Society, 357-384.
13. Haas, M. Mittnik, S. & Paolella, M. S. (2004a). A New Approach to Markov-Switching GARCH Models. Journal of Financial Econometrics, 2, pp. 493-530.
14. Pastpipatkul, P., Yamaka, W., & Sriboonchitta, S. (2016). Analyzing financial risk and co-movement of gold market, and indonesian, philippine, and thailand stock markets: dynamic copula with markov-switching. In Causal Inference in Econometrics (pp. 565-586). Springer International Publishing.
15. Patton, A. J. (2006). Modelling asymmetric exchange rate dependence. International economic review, 47(2), 527-556.
16. Da Silva Filho, O. C., Ziegelmann, F. A., & Dueker, M. J. (2012). Modeling dependence dynamics through copulas with regime switching. Insurance: Mathematics and Economics, 50(3), 346-356.